

Olber's Paradox

“Why is the sky dark at night?” It shouldn't be. According to the following logical argument, it should be quite bright.

1. The apparent intensity of a light source decreases with the square of its distance from the observer. Assuming no interstellar dust absorption, this is true. Lumens received from a star will vary inversely as the square of the distance to that star.
2. Assuming the distribution of stars is uniform in space, then the number of stars at a particular distance, r , from the observer will be the number that are located within a thin spherical shell whose radius is r , whose area is $A = 4\pi r^2$ and whose radial thickness is dr . So at radius r , there are $k4\pi r^2 dr$ stars.
3. Therefore, at each and every possible radial distance, r , the amount of light that gets to us from that shell should be both directly proportional to the radius squared (the number of stars) and inversely proportional to the radius squared (they get apparently dimmer with distance).
4. These two effects cancel each other. So, every spherical shell of radius r should add the same small additional amount of light. Thus the total lumens received will

$$\text{be: } Lumens = 4\pi k \int_0^{\infty} \frac{r^2}{r^2} dr = \infty .$$

5. Ergo: In an infinite universe, when we sum (integrate as in step 4 above) the light coming from all the infinite and uncountable number of possible radial distances, r , the sky should be infinitely bright.

But the night sky is not infinitely bright. It's dark. Why?

The resolution of this paradox can be achieved by realizing that there are only approximately 8400 stars in our night sky that we can see with our naked eyes. We do not see the others; they are too dim. Yes, yes, Carl Sagan may have talked about *millions and billions* of stars – but we can only see about 8400 with our unassisted eyes. Carl was well known for his tendency to exaggerate. We get the impression of millions and billions when we look up at the Milky Way, but we can see only about 8400 individual stars – that's it – and that's under ideal conditions.

Down through the ages we have come to categorize stars into various brightness classes called “magnitudes”. Quite arbitrarily we say that the dimmest stars we can see are of class “magnitude 6”. These stars put out only 1% of the light (measured in lumens) of those in the (bright) “magnitude 0” class. Therefore we see that each time we go up one step in brightness class (say from magnitude 4 to magnitude 3) we go up in light output $100^{1/5} = 2.512$ times. (Because $2.512^5 = 100$).

Of course a star can appear bright because it *is* bright, but it also might be very close to us. There is a well-known relationship between distance and the apparent magnitude of a star. For example, if we move a (very bright) star of 0th *absolute* magnitude¹ to a distance of 51.8 light-years, its *apparent* magnitude will go down to

¹ If a star is 10 parsecs = 32.6 Light Years distant, and it *looks* like a 1st magnitude star, we say it has an *absolute magnitude* = 1. Ten parsecs is the “standard distance” at which we measure absolute magnitude.

1.0. Humans cannot see any star whose magnitude is higher (less luminous) than about 6.4. The 200 inch Hale telescope at Mt. Palomar can see down to about magnitude 23 or so.

Of course, some stars are much brighter than absolute magnitude 0 or 1.0 and thus would be visible farther out than around 600 LY. But, many are much dimmer too, so as an overall approximation let us consider the average star. If it is farther away than 600 LY, we humans cannot see it (*at all*). So it might as well *not be there at all*.

To our naked eyes, then, the total light in our night sky is not affected by much of anything that is dimmer than magnitude 6.4 (typical stars farther away than around 600 LY). The light sensing nerve endings in our eyes are nonlinear. There is a lower boundary value such that light weaker than that will not produce a signal to our brain that says, "I have seen something." Even for the blue-white giant stars whose absolute luminosity puts them at -10 or -12 (much brighter than absolute magnitude 1.0), there exists some finite distance beyond which they too become invisible to us – their apparent magnitude slips down beyond the 6.4 limit. So a distant horizon exists beyond which we cannot see any stars.

There *are* a very few vastly distant objects that we can see with our naked eyes such as the Great Andromeda Galaxy (M 31). It is over 3 million LYs away. But it is such a concentrated collection of stars and plasma that it looks to us about as bright as a single magnitude 4 star. There are bigger and brighter galaxies than M 31 out there, but they are so distant that their apparent magnitude is greater (dimmer) than 6.4, so we can't see them.

The point is this – it should now be obvious – the infinite sum described in steps (4-5), above, is incorrect. The sum *stops* (is truncated) at a radial distance of some 600+ light years for the typical star (and somewhere beyond that for even the brightest ones). There is an upper limit on the absolute brightness of a single star; there is no such thing as an infinitely brilliant star. So there is a finite upper limit to the integration process described above. It doesn't go out to infinity.

It may also help to remember that the human eye is different from photographic film or a CCD chip in that it does not integrate over time. The longer we expose a photographic plate to starlight the brighter the image becomes. (There is a limit even to this process in film due to what is called *reciprocity failure*.) Theoretically the longer we expose our CCD camera chip, the brighter the image (deeper into space we can see). This is not true for the human eye. Humans can stare up at the dark sky all night long and not see anything they didn't see after the first few minutes. Things don't get brighter for us the longer we look at them. We can see the 8400 or so stars that we can see, and all the zillions of others might as well not be there *at all* as far as our ability to see them is concerned.

Olber's Paradox is not a paradox if you look at it correctly. It is simply an error. It results from a mathematical analysis being applied incorrectly to a real world phenomenon. A mathematician might say, "They got the upper limit on the integral in step 4 wrong." There is only one right answer – the night sky is dark.

I think it was the eminent mathematician Groucho Marx who said, "Who 'ya gonna believe, me or your lying eyes?"